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Operations of a Game Show: Dynamic Programming Analysis of Family Feud

ABSTRACT

This paper presents the dynamic programming analysis of Family Feud, a TV game show. It analyses the events where the responses provided by a family do not match the answers on the scoreboard (strikes), thus leading to termination of the family's chance at answering the question and thereby creating an opportunity for the opposing family potentially to “steal" points accrued. We obtain the optimal strategies to maximize the expected earnings for playing the game in the face of events such as strikes and steals.

KEYWORDS: Decision Support Systems, Group Decision Making, Dynamic programming and Stochastic analysis.

INTRODUCTION

Games provide a popular avenue to stretch human imagination and engage strategic thinking. Games have been used to represent whimsical situations that are not often a universal part of everyday life; industry and banking, tactical events, exit from uncertain events, and thousands more that have been an integral part of human culture. The rules of a game define the strategic thinking skills that will be most useful in achieving the most desirable outcomes. These games range from ancient ones that are still popular, such as chess, to modern video games, such as Civilization. One of the primary outcomes of games is the uncertainty of the outcome – the uncertainty is part of the appeal that engages users through the elements of surprise and attention. This outcome of uncertainty in games has been the subject of significant research through the avenue of stochastic programming, which studies decision making under uncertainty. Stochastic programming is an integral part of research in multiple business applications – some of them include risk management, liability planning, portfolio optimization, asset management, production planning and logistics. In this paper, we investigate the crucial events in Family Feud, a TV game show, and present optimal strategies for winning the game using dynamic programming and stochastic analysis. Family Feud is a popular TV game show in which two families compete to provide the most popular answers to survey questions.

The use of dynamic programming strategies to analyze TV game shows has been presented in Perea and Puerto (2006), Cochran (2001), Rump (2001), Thomas (2003), Korsos and Polson (2013), Hartley and Walker (2006), and Trick (2001). Unlike Who Wants to be a Millionaire, where the contestant has to pick a correct option out of four listed options, Family Feud does not seek to encourage contesting families to provide the correct answer; only the most popular answers are sought. This emphasis on the most popular answers is part of its wide appeal; contestants need not be experts in any domain. The show routinely appears on lists of most popular TV game shows since its inception in 1976. As of June 2015, Family Feud claimed the number one spot in TV syndication ratings.

Rules of the Game: Each family has five members and a family leader. The TV show is organized into two parts: the Family Feud (FF) game and the Fast Money (FM) round. The family that wins the Family Feud game gets an opportunity to send two of its members to play the Fast Money round.

Family Feud Game Structure: The game is organized as a series of four rounds. The host asks opposing members of the two families to answer a question in each round.

Questions: The questions are survey questions that have been administered before the game commences, and the responses collected have been classified into categories based on the number of respondents in a particular category. The questions are based on popular culture and daily activities, and as a consequence the answers reflect the most popular responses from the group of people surveyed. For example, the question “Name a member of the Muppets clan?" might yield responses such as Kermit, Miss Piggy, Fozzie Bear, Gonzo, Rowlf, as well as other answers. The member who presses the buzzer first gets a chance to answer the question. If the answer provided is one of the survey answers, it is listed on the scoreboard along with the answer and the number of respondents who provided the same answer. The host then asks the opposing member the same question. If the opposing member's answer is one that is the more popular of the two, that family gets a chance to decide if they want to provide other answers to that question (play the game) or pass the question back to the family that originally asked the question (pass the game).

Every member of the participating family is then asked the same question, and their responses are checked to determine if they belong in the set of responses obtained from the survey. Each response that matches any of the survey responses gathers points. So, in response to the Muppets question above, if a family member answered Fozzie Bear, and Fozzie Bear was the response provided by 27 individuals who answered this question in the survey, the family gets 27 points added to its score.

There are two scenarios in which a family's opportunity to answer a question is terminated: (1) provide three responses that do not match the survey responses (strikes), or (2) provide responses that all match the survey responses.
Three strikes: If a family member fails to provide an answer that is contained in the survey responses, it is treated as a strike, like in baseball. Each family can gather at most three strikes before the question is then passed to the opposing family. The opposing family then has a chance to cooperate and choose one response. This response is provided by the family leader. If this response matches one of the unanswered survey responses, the family can get all the points for that round, including the points gathered by the opposing family for answering the question before they lost control of the question due to three strikes. This is referred to as “stealing the game" in Family Feud parlance. If the one response provided by the Family Feud leader does not match any of the unanswered survey responses, the points are not “stolen," and the family that first had control of the questions retains all the points that they gathered for answering the question.

All matches: If the members of a family provide all the survey responses for a question, they win that round, and the opposing family does not get a chance to answer that question.
Every round ends in either of the above two methods: stealing a question or providing all answers to the question. In the event that all answers are not provided, the host finishes the round by announcing all the answers that were not provided by either family. The game then moves to the next round.

Four rounds of play follow the same structure with the only difference being that the questions become progressively more valuable. Whereas responses in the first two rounds are worth one point each, responses in the third and fourth round are worth two points and three points each respectively. The first family to reach or exceed 300 points wins the Family Feud game. If neither family gathers 300 or more points, there is one final question that serves as the tie breaker. Opposing members are required to answer the tie-breaker question and provide only the most popular response. The family member who provides the most popular response procures a win for the family and hence a chance to move forward to the Fast Money Round.

Fast Money: The family that wins the Family Feud game has an opportunity to play the Fast Money round. In this round, two members of the winning family are asked the same five questions. The first member gets 20 seconds to answer the questions while the other member is secluded backstage, completely unaware of the questions or the responses given by the first family member. When the first member finishes, the previously secluded family member is granted a chance to answer the same five questions within 25 seconds. The responses offered by the two family members must vary, and hence the second member is offered an additional 5 seconds to answer the question and change a response if it has already been provided. The goal of this round is for the two members of this family to obtain 200 or more points. If they gather less than 200 points, their total earnings for the game are the points obtained in the Fast Money game multiplied by 5. If they gather 200 or more points, their earnings for the show equal $20,000. In either case, the family gets a chance to come back for another Family Feud game with a different family. Thus, winning the Family Feud game guarantees a family two opportunities for increased earnings: a potential winning of $20,000 in the Fast Money round and the chance to participate in another Family Feud game. A winning family can continue to play the Family Feud game four more times (for a maximum allowed total of five games per family) with the constraint that a family must win a game to get a chance to play another game. If a family wins five Family Feud games, they also win a car.

The methodologies outlined in this paper provide insights into the often random-appearing nature of the game. Although the questions are based on popular culture and quotidian activities Bialik (2008), the highly random answers provided by the surveyed population reflect a key feature of the playing strategy: the players are not looking for the correct answer; they seek only to identify the most popular answer. Indeed, multiple instances abound of events in which correct answers have been provided, but they do not match the scoreboard answers. Thus, an important Family Feud game component is not just the ability to provide popular answers that are answered by majority of the survey-takers but also the ability to think of answers that are provided by a few people (at least two) to match the scoreboard responses. In this paper, we present a stochastic analysis of these events based on the theory of estimation of rare events using the Good-Turing estimate and show how to maximize the chances of winning a round as well as the chances of accruing 300 points faster than the opposing family.

The remainder of this paper is organized as follows. Section 2 presents an analysis of the payoffs possible in the Family Feud game show using a mathematical framework. Section 3 presents a stochastic analysis of the events of strikes, steals and consequent wins or losses using the Good-Turing model to estimate the probability of providing an answer that does not match the scoreboard answer. Section 4 presents the results of our analysis and simulations. Section 5 presents conclusions drawn from this analysis.

MATHEMATICAL FRAMEWORK FOR THE PAYOFF ANALYSIS

We identify three distinct earnings outcome cases in the game.

Case 1: A family wins 5 games and 5 Fast Money rounds. The earnings in this case equal $100,000 (a win in a Fast Money round is equal to $20,000) and a car (since they won five games). This is the best-case scenario and represents the maximum possible earnings for the game.

Case 2: A family wins 5 games and n_a Fast Money rounds, where $n_a < m_a$. In this case, the family's earnings are a car (since they won five games) and the earnings from each of the Family Feud games added to the earnings from the Fast Money rounds ($20,000 times the number of Fast Money rounds that were won).

Case 3: A family wins ma games, where $1 \leq m_a$ and n_a Fast Money rounds, where $n_a \leq m_a$. In this case, the total earnings equal the earnings from each of the Family Feud games added to the earnings from the Fast Money rounds ($20,000 times the number of Fast Money rounds that were won).

Case 4: A family does not win its first game and therefore does not receive any earnings from the game. This is the worst-case scenario and represents the least earnings possible ($0) from the game.

We now provide a mathematical framework for the first three cases described above with a non-zero earning potential.

Let m_a be the number of games played, and let n_a be the number of Fast Money rounds that have been won, where $n_a \leq m_a$. Let w be the winnings.

Bounds on n_a and m_a: A family has a maximum of 5 chances of competing. Therefore, $1 \leq m_a \leq 5$. Furthermore, the number of Fast Money Rounds is represented by $0 \leq n_a \leq 5$.

Case 1 (Best case scenario): $m_a = 5$, $n_a = 5$

$$w = C + 100 * 10^3 \tag{1}$$

The first term on the right hand side of this equation represents the winning of a car from five wins of Family Feud games. The second term represents five winning Fast Money rounds, each amounting to $20,000.

Case 2: $m_a = 5$; $0 \leq n_a \leq 4$

$$w = C + \sum_{i=1}^{m_a - n_a} a_i + n_a (20 * 10)^3 \tag{2}$$

The term $\sum a_i$ represents ma games that were played, of which only n_a resulted in winning Fast Money rounds. We call the winnings represented by the term the “moderate earnings." Thus, the term $m_a - n_a$ in the upper limit of the summation represents the number of games that only resulted in moderate earnings.

The second term in this equation given by $\sum a_i$ represents the total earnings from all Family Feud games that were won but did not result in a winning Fast Money round. The term represents the earnings from a single winning Family Feud game. The third term in the equation represents earnings from the winning Fast Money rounds.
Case 3: $1 \leq m_a < 5$, $n_a \leq m_a$

$$w = \sum_{i=1}^{m_a - n_a} a_i + n(20 * 10^3) \tag{3}$$

Bounds on the number of Family Feud (FF) points: For a family to make it to the Fast Money round, they have to gather at least 300 points. Thus, 300 points represents the lower-bound on the points. The typical survey size for collecting responses to survey questions in Family Feud games is 100. Assume that a family provides all the responses for a question and thereby matches all 100 answers provided in the survey. Assume also that this pattern continues for all four rounds of the game. As a consequence, the total number of points won by the family equals 700 (100 for each of the first two rounds, 200 for the second and 300 for the third) and represents the upper bound on a family's points in the Family Feud game.

STOCHASTIC ANALYSIS OF FAMILY FEUD

For any given round, the maximum number of points that can be earned is 100. However, the answers that are displayed on the scoreboard represent only those that obtained at least 2 responses. Hence, the total number of points represented on the board is less than or equal to 100. The survey responses represent the most popular responses to a question and as a result may not necessarily reflect the “correct" answer to the question.

Probability of a strike: When a family provides an answer that is not on the board, it is called a strike. A strike can mean one of three things:

1. The answer is irrelevant. (For example, in response to the question, “What is the most popular food item in a Thanksgiving menu?” , a response of “spoon" would be deemed irrelevant.)
2. The answer is relevant, but only person in the survey provided the same response. For the rest of the paper, we will refer to such answers as 1-response answers.
3. The answer is relevant, but no one in the surveyed population provided that response.

Since providing irrelevant answers reduces the payoff for participating in the game, family members cooperate and come up with answers that are relevant. The use of option #1 is more conducive to receiving a strike, and the strategy used by families is that of cooperation to provide popular answers to the question. Option #3, on the other hand, is less appealing. Family Feud surveys are carefully engineered to follow standard survey practices to try to ensure that the sample is representative of the US population Bialik (2008). Consequently, we do not consider the probability that the answers on the show do not match the answers provided by the sample population. In this paper, we focus on option #2. Choosing option #2 also reduces the sample space of probable answers. Option #1 uses a space that is infinite since the number of answers can be all the words and phrases possible in the English language. However, in practice, families provide answers that are most relevant to the question, and thus option #2 creates a finite space for probable answers.

Within the finite set of probable answers, we focus only on answers that were provided by at least one person in the sample. Thus, in a round that has n answers to a question on the board, at most $n-1$ questions can have two responses each. The probability of getting a strike can then be framed as the probability of picking an answer that was provided by only one other person in the sample. Let p represent the total number of 1-response answers.
Since p represents events that are least likely, we use the Good-Turing theorem to find the probability of their occurrence. The Good-Turing theorem Good (1953) uses the probability of events that have occurred to estimate the probability of events that have not occurred.

Conventional maximum likelihood estimator [MLE] solutions to this problem would predict that the probability of answering “chair" to the question “What is the most popular Thanksgiving food?" is 0, since the set space is infinite. However, in practice, the set space is finite and can be reduced to relevant 1-response answers. Of these, the answers that were provided by at least two persons in the sample (2-response answers) are displayed on the board. The Good-Turing estimate for calculating the probability of a 1-response answer, given that it appeared r times in the sample is given by

$$P(r) = \frac{1}{N}(r+1)\frac{N_{r+1}}{N_r} \tag{4}$$

where the answers are assumed to be independent in the set space and N_r denotes the number of item types that appear r times in the sample.
Thus, for a question that has n answers provided by at least two people in a sample of 100 people, the probability of picking a 1-response answer (and hence not on the scoreboard) is

$$P(r_1) = \frac{2(n-1)}{p}\frac{1}{100} \tag{5}$$

After 1 strike, the number of answers provided by only person is reduced to $p-1$. The probability of two strikes is thus given by

$$P(r_2) = \frac{2(n-1)}{p\text{-}1}\frac{1}{100} \tag{6}$$

The probability of three strikes (and hence a steal) is given by:

$$P(steal) = p(r_3) = \left(\frac{2(n-1)}{p-2}\frac{1}{100}\right) \tag{7}$$

Post-steal Options

Assume that at the end of three strikes, the family has gathered x points after answering n_1 of the total n answers. There are two options for the round to progress after a steal.
a. The opposing family provides an answer that is on the board worth y points: The probability of a steal and a win by the opposing family (loss by the current family) is given by

$$P(s_1) = \left(\frac{2(n-1)}{p-2}\frac{1}{100}\right)\left(\frac{1}{n-n_1}\right), \tag{8}$$

where $n_1 < n$. (If $n_1 = n$, the family wins the round).
The total payoff for the current family is 0, while the opposing family gets $x+y$ points.

b. The opposing family provides an answer that is not on the board: The probability of a steal and a loss by the opposing family (i.e., a win for the current family) is equivalent to the probability of a steal from the current family and a choice of a 1-response answer by the opposing family.

$$P(s_2) = \left(\frac{2(n-1)}{p-2}\frac{1}{100}\right)\left(\frac{2(n-1)}{p-3}\frac{1}{100}\right). \tag{9}$$

The total payoff for the current family is x, while the opposing family gets a payoff of 0 points.

RESULTS

Figures 1 and 2 represent the probability of a steal and loss for the current family, where represents the total number of answers that had only one respondent. The graphs are obtained by varying the value of n in equation (8) from 4-10 (the number of answers on the board) for rounds 1-4. The value of n_1 varies from 1 to $n-1$, since at least one answer has been provided by the time the family takes control of a question. From Figures 1 and 2, we see that the probability of a loss for the current family in each round increases with the number of answers that have been provided. An increased number of answers that have already been provided reduces the space of answers that are not yet on the scoreboard, and hence the probability of picking an answer from the reduced space is lower and leads to a higher probability of loss. This also explains the higher probability of loss in rounds with more answers.

Figure 1: Probability of steal and loss for current family, $p=5$

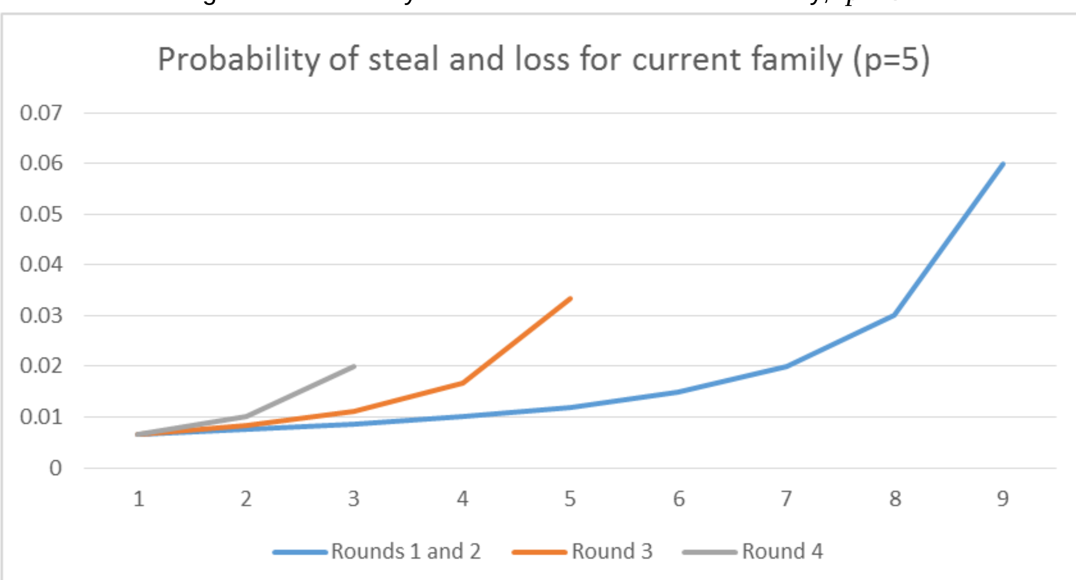

Figure 2: Probability of steal and loss for current family, $p = 10$

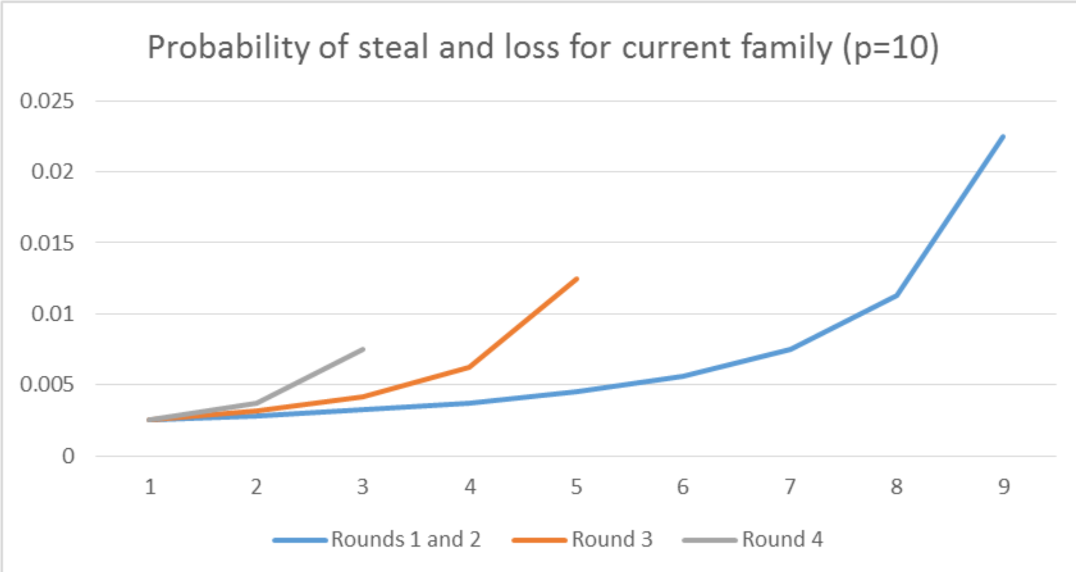

Figure 3: Probability of steal and win for the current family

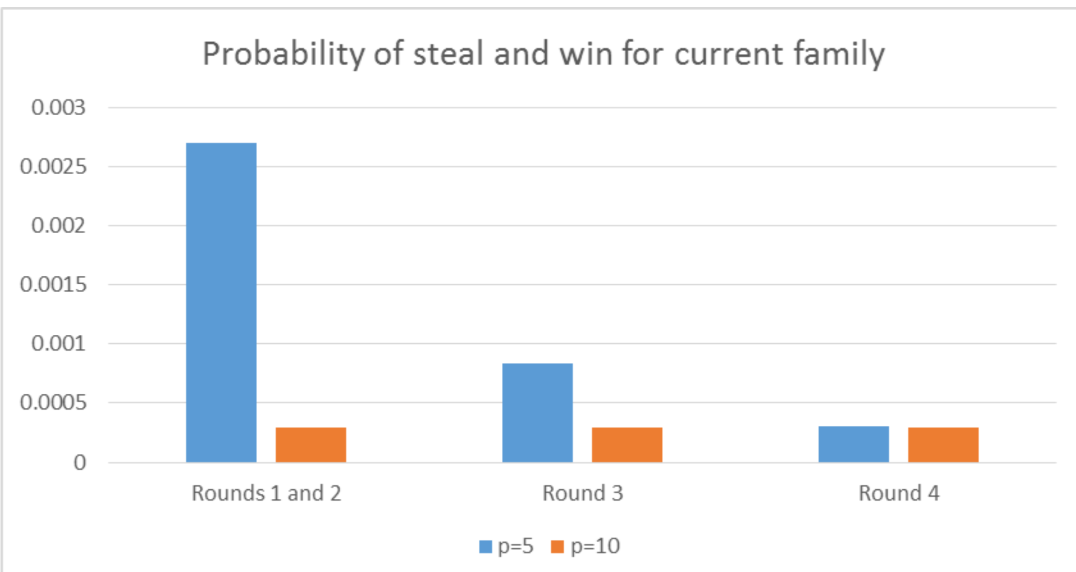

Figure 3 is obtained for two different values of p in equation (8). However, since the opposing family has only one chance to answer the stolen question, p is independent of n_1. We see that the probability of a steal and win for the current family is much lower than the probability of a loss. This can be attributed to the dependence of $P(s_2)$ solely on the Good-Turing estimates of choosing a 1-response answer, unlike $P(s_1)$, which depends on the Good-Turing estimate and the MLE.

The Good-Turing estimate for the probability of choosing a 1-response answer by the opposing family (and hence a win for the current family) is lower than the probability of choosing a scoreboard answer due to the lower incidence of 1-response answers.

Table 1: Payoff matrix in the event of a steal/win and steal/loss for four rounds

	Rounds 1 and 2	Round 3	Round 4
Steal and Win	x_1	x_2	x_3
Steal and Loss	0	0	0

Table 2: Payoff matrix of accumulated points for opposing family

	Rounds 1 and 2	Round 3	Round 4
Steal and Win	0	0	0
Steal and Loss	$x_1 + y_1$	$x_2 + y_2$	$x_3 + y_3$

Tables 1 and 2 show the payoff matrices of points obtained during the four rounds of the Family Feud game. The points obtained by the current family and opposing family in the four rounds are shown in the event of a steal/win and steal/loss scenario. An analysis of this table shows that the highest payoff in terms of accumulation of points occurs when the opposing family steals the question and provides an answer that is on the scoreboard. It also represents the least payoff for the current family since they lose all the points they had obtained.

Strategies for Winning the Game

For a family to reach the required minimum of 300 points to win the FF game, multiple strategies exist:

a. Provide all answers to a question (low probability) and hence win every round. Assume there are u unique words in the English language out of which v match the responses to a question in a given round. The value of v will be a maximum of 10 (rounds 1 and 2), 6 (round 3) or 4 (round 4). As of January 2014, the value of u is around a million according to Monitor (2014). Thus, the probability of winning all rounds by providing all responses to the questions is

$$P\left(win_{game}\right) = \left(\frac{10}{10^6}\right)^2 \left(\frac{6}{10^6}\right)\left(\frac{4}{10^6}\right). \tag{10}$$

b. Provide the top half of the responses, accumulate three strikes (steal) but the opposing family loses, thereby resulting in the current family's winning every round (Table 2)

c. Winning fewer than 4 rounds but still getting a cumulative of 300 points (Table 3)

d. Winning fewer than 4 rounds and using the tie breaker (not provided to avoid redundancy)

To obtain the probability of choosing the most popular responses and thereby obtaining the most number of points, we find the probability of choosing the top half of the responses on the scoreboard. Assume a family has all the answers but does not know the frequency of those answers in the sample. The problem of identifying the top half of the answers in every round is equivalent to the problem of finding a match.

For v answers on the score board, the probability of getting the top half is given by:

$$P_{half} = \left(\frac{1}{v}\right)\left(\frac{1}{v-1}\right)\left(\frac{1}{v-2}\right)\left(\frac{1}{v-3}\right). \quad (11)$$

Tables 3 and 4 show the probabilities of winning a maximum of 4 rounds, with a win denoted by W and a loss denoted by F. Table 3 shows the probabilities when only one answer provided during the buzzer round is matched on the scoreboard whereas Table 4 shows the probabilities when both the answers provided during the buzzer round are matched on the scoreboard. Table 5 shows the ways of winning fewer than 4 rounds. (SL stands for steal and a loss, SW stands for steal and a win, and W stands for winning a round without any steals.) A round may be won by providing all answers or by steal and win (when the opposing family does not provide a scoreboard answer). The probability of winning a round without any steals is given as:

$$P(win_{round}) = \left(\frac{10}{10^6}\right) + \left(\frac{10}{10^6}\right) + \left(\frac{6}{10^6}\right) + \left(\frac{4}{10^6}\right). \quad (12)$$

The probability of not winning a round is given by $P(SL) = P(s_1)$ as indicated in equation (8). The probability of steal and win is given by $P(SW) = P(s_2)$ as indicated in equation (9).

Figure 4: Average Earnings for a family in Family Feud (Winnings for the term $\sum_{i=1}^{m-n} a_i$)

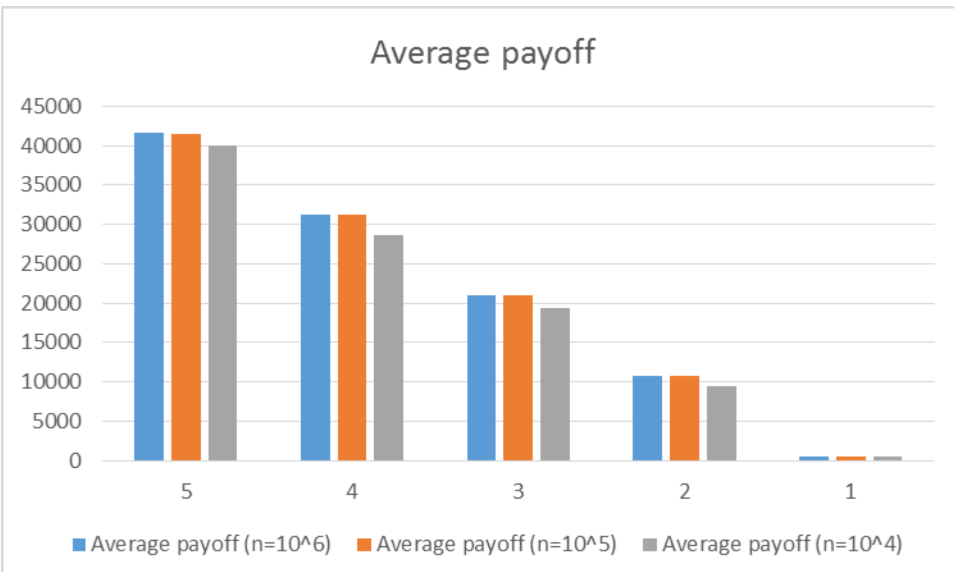

Figure 4 shows the average earnings for a family participating in the FF game show. The number of FF games that a family can win is chosen randomly between 1 and 5 to signify that a family can win at most 5 games in the show. The number of FM rounds are varied randomly between 0 and the number of winning FF games for a family. The average payoff is shown for three different levels of simulations of the algorithm (10^6, 10^5, and 10^4). In each case, we see that the payoff is increased with the number of FF games that a family plays.

Table 3: Matrix of combinations of wins and losses in four rounds and corresponding probabilities when one answer from the buzzer round has been matched on the scoreboard, given that there are no steals.

Winning Rounds	Combinations Possible	Outcome Probabilities
4	WWWW	$\left(\frac{1}{840}\right)^2\left(\frac{1}{60}\right)\left(\frac{1}{6}\right) = 3.93676E-9$
3	WWWF, WWFW, FWWW, WFFW	$\left(\frac{1}{840}\right)^2\left(\frac{1}{60}\right)\left(\frac{5}{6}\right) + \left(\frac{1}{840}\right)^2\left(\frac{59}{60}\right)\left(\frac{1}{6}\right) + \left(\frac{839}{840}\right)\left(\frac{1}{840}\right)\left(\frac{1}{60}\right)\left(\frac{1}{6}\right) + \left(\frac{839}{840}\right)\left(\frac{1}{840}\right)\left(\frac{59}{60}\right)\left(\frac{1}{6}\right) = 0.00019936$
2	WWFF, WFFW, WFWF, FFWW,	$\left(\frac{1}{840}\right)^2\left(\frac{59}{60}\right)\left(\frac{5}{6}\right) + 2\left(\frac{1}{840}\right)\left(\frac{839}{840}\right)\left(\frac{59}{60}\right)\left(\frac{1}{6}\right) + 2\left(\frac{1}{840}\right)\left(\frac{839}{840}\right)\left(\frac{1}{60}\right)\left(\frac{5}{6}\right) + \left(\frac{839}{840}\right)^2\left(\frac{1}{60}\right)\left(\frac{1}{6}\right) = 0.00319511$
1	WFFF, FWFF, FFWW, FFFW	$2\left(\frac{1}{840}\right)\left(\frac{839}{840}\right)\left(\frac{59}{60}\right)\left(\frac{1}{6}\right) + \left(\frac{839}{840}\right)\left(\frac{1}{60}\right)^2\left(\frac{5}{6}\right) + \left(\frac{839}{840}\right)^2\left(\frac{59}{60}\right)\left(\frac{1}{6}\right) = 0.17706301$
0	FFFF	$\left(\frac{839}{840}\right)^2\left(\frac{59}{60}\right)\left(\frac{5}{6}\right) = 0.81749455$

Table 4: Matrix of combinations of wins and losses in four rounds and corresponding probabilities when both answers from the buzzer round have been matched on the scoreboard, given that there are no steals.

Winning Rounds	Combinations Possible	Outcome Probabilities
4	WWWW	$\left(\frac{1}{360}\right)^2\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) = 1.6075E-07$
3	WWWF, WWFW, FWWW, WFFW	$\left(\frac{1}{360}\right)^2\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{360}\right)\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{359}{360}\right)\left(\frac{1}{360}\right)\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{360}\right)\left(\frac{359}{360}\right)\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) = 0.001388889$
2	WWFF, WFFW, WFWF, FFWW, FWWF, FWFW	$2\left(\frac{1}{360}\right)\left(\frac{359}{360}\right)\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{359}{360}\right)^2\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{359}{360}\right)^2\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) = 0.023645833$
1	WWFF, WFFW, FFWF, FFFW	$2\left(\frac{1}{360}\right)\left(\frac{359}{360}\right)\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{359}{360}\right)^2\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) + \left(\frac{359}{360}\right)\left(\frac{23}{24}\right)\left(\frac{1}{2}\right) = 0.497233454$
0	FFFF	$\left(\frac{359}{360}\right)^2\left(\frac{1}{24}\right)\left(\frac{1}{2}\right) = 0.476508327$

In Figures 1, 2 and 3, the probability of steal and loss/win is lower for a higher value of p, signifying that the greater the number of 1-response answers for a question, the lower the probability of correctly providing all responses to a question becomes. Figure 4 represents the winnings table for the moderate earnings term, where m represents number of games played (1-5), and n = number of games in which a family wins Fast Money

Table 5: Matrix of possible outcomes for winning fewer than 4 rounds and computations of associated probabilities

Number of Wins	Possible Outcomes	Computation of Probabilities
0	SL	$P(s_1)$
1	(SW or W)	$P(s_2) + P(win_{round})$
2	(SW or W) and (SW or W)	$[P(s_2) + P(win_{round}]^2$
3	(SW or W) and (SW or W) and (SW or W)	$[Ps_2 + P(win_{round})]^3$

CONCLUSIONS

In this paper, we presented an analysis of the events in the Family Feud TV game show that are critical to advancement in the games. These events are answers that do not match the scoreboard answers (strikes), steals (the event of three strikes) and the event of reaching 300 points first. We show the probability of winning a round despite a steal and present the optimal strategies for accumulating points. Further, we present the simulation results for estimating the average earnings of families who appear on the show no more than 5 times. The results of our analysis show that the probability of a loss for the current family in every round increases with the number of answers that have been provided to match the scoreboard answers. We also found that the probability of a steal and win for the current family is much lower than the probability of a loss due to the high entropy of the survey responses. The uncertainty of survey answers matching the family's responses is reflected in the low probability of providing all responses to a question. We see that the greater the number of 1-response answers for a question, the lower is the probability of correctly providing all responses to a question. For a family that has control of the question, the optimal strategy for winning a round is to prevent the event of the question's being stolen. On the other hand, the optimal strategy for the opposing family is to steal a question and provide an answer that matches one of the unmatched scoreboard answers. This results in stealing points and accumulating additional points through a successful steal. We also show that the fastest way to reach a minimum of 300 points is to win at least two rounds of the Family Feud game. This ensures a chance at playing the Fast Money round for a potential earning of $20,000 and a chance to compete again. The dynamic programming analysis of the expected earnings in the Family Feud game shows that the expected earnings are directly proportional to the number of games that a family plays, with the highest earning potential achieved by playing 5 games. A stochastic programming approach to the study of games such as Family Feud has important insights for areas in business such as production planning, operational planning, manufacturing scheduling and energy distribution. Modeling and analyzing stochastic events such as game shows present opportunities for applications of decision theory in non-traditional business areas, secondly they offer valuable insights into the operations of high-risk events that could be applied to traditional business applications. The work in this paper highlights the application of stochastic programming to decision-making in Family Feud, and presents the randomness inherent in multiple areas of the game.

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